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Probability learning in mathematics using augmented reality: impact on student's learning gains and attitudes

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ABSTRACT

The development of Augmented Reality technologies has enabled students to learn in an environment that combines learning resources from the real and digital world. This paper integrates three mobile Augmented Reality-based applications into a series of mathematics lessons on probabilities in a junior high school. This paper aims to examine how mobile AR-based learning applications affect students' learning gains through ANCOVA. Moreover, students' attitudes towards the applications are discussed. The participants are 68 junior high school students. The experimental results show that the mobile AR-based applications would be helpful for students' learning gains in the topic of probability. Students' attitude towards the AR applications in this series of lessons was positive. Overall, it was found that mobile AR-based learning applications have positive effects on junior high students learning mathematics.

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Augmented reality; mathematics learning; probability learning; math education; application in math education

1. Introduction

The virtual learning environments have brought a brand-new experience to learners and got attentions from many researchers. Chittaro and Ranon (2007) reviewed many cases of 3D technologies in learning and education, then they found that the learners in proper-design virtual environments could use less cognitive effort to get better experiences and knowledge than traditional educational environments. According to Marriott (2005), doctors who play simulation video applications 3 h per week made fewer mistakes and completed operations faster than those who had never played simulation applications. 3D virtual environments like Second Life was used in many schools as virtual classroom tool (Warburton, 2009). It could provide educational opportunities for more people. To a considerable extent, it could also reduce the "Digital Divide" by helping students who have no access to good education.

Augmented Reality (AR) is generally considered to be an extension of generalised Virtual Reality (VR). This technology was defined in computer science as having three essential features: having virtual elements overlaid over the real world, real-time interaction and being registered in 3D (Azuma, 1997). In shortly, AR mixes virtual objects with the real world. Users could interact with virtual 3D objects in the real environments. An ideal AR-based learning environment vividly enables 3D representation of invisible and difficult-to-visualise events, facilitates the realisation of topics and provides an understanding of subject that students find difficult. AR could be used with instructional strategies like discovery, cooperative and collaborative learning (Ibáñez & Delgado-Kloos, 2018).

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However, it is worth studying whether a modern information technology is beneficial to all audiences in education. Many studies have tried to employ new technologies or environments in learning to provide richer contents. But people should investigate the effects of these technologies or environments on different students with diverse personal features (Shih, Chu, & Hwang, 2011). A lot of studies have integrated AR in education but few of them paid attention to diverse learners. In this study, a set of mobile-based AR learning applications were developed for students learning the concepts and applications of probability in Mathematics. The knowledge point comes from real life, but it is abstract and difficult to visualise. This study focuses on two aspects: whether AR software can promote the learning gains of junior high school students studying probability and statistics and whether students are willing to accept such learning tools in this context.

2. Related works

2.1. Using technologies to teach probability and statistics

Probability and statistics are important subjects in K-12 education. In the United States, Statistics and Probability is an independent conceptual category in the high school standards (National Governors Association, 2010). In Asian countries, Statistics and Probability is an obligatory learning content in Mathematics in high schools. Some of the most recent studies on the teaching and learning of probability and statistics at different levels of education are briefly discussed below.

In contemporary studies on elementary and secondary mathematics classes, a series of research focuses on how to help students to reason and understand probability and statistics. Hawkins and Kapadia (1984) used psychological and pedagogical ways to research children's conceptions of probability and statistics. They have found that young children's intuitive notions of probability are typically subjective and incoherent. Meanwhile, children's subjective judgements may well conflict with the formal probability axioms. Based on the experiences from psychologists, educators, and statisticians, a large proportion of students, including those attending college, do not understand some basic statistical concepts they have learnt, (Garfield & Ahlgren, 1988). The percentage of secondary school students who understand the concept of key concepts, such as independent events, was much lower than the test results from the National Assessment of Educational Progress (Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993). Those findings revealed that students were not doing well in probability and statistics. Many students' understandings of the probability concepts were based on their preconceived meanings, not on the established conventional probability meanings (Kazima, 2007). Garfield and Ben-Zvi (2007) reviewed research on teaching and learning of statistics, focusing on students' comprehension of concepts and the well-designed sequences of learning activities with appropriate technologies. They concluded that this could help learners improve their reasoning and understanding. However, most of the research on the integration of technological tools into statistics and probability courses have been conducted in college settings rather than at the secondary school level. In the secondary level, graphing calculators were the most commonly used tool students chose to compute, transform, collect, analyse, visualise, and check data in mathematic courses (Doerr & Zangor, 2000). Microsoft Excel was widely used as a supplement in high school statistics course. Christensen and Stephens* (2003) compared students using Excel to students using traditional graphing tools. The results showed that the Excel group outperformed the traditional group in the unit tests. Besides Excel, Lesh, Middleton, Caylor, and Gupta (2008) used Fathom as a supplement to teach data modelling concepts to middle school students and their teachers. Their study indicated that Fathom made data modelling more expressive and intimate, aiding students in the development of more profound and more applicable mathematical understanding. Finally, both Hudson, Cross, Lee, and Rapacki (2012) and Lee (2014) illustrated Tinkerplots, a dynamic statistical software, allowed students to efficiently organise large data sets and made useful conclusions about the meaning of the data.

Among the studies that used appropriate technological tools in college settings, Smith (2003) evaluated three computer-assisted simulations developed for use with SPSS and Microsoft Excel, aiming at reinforcing and enhancing students' understanding of sampling distributions and related concepts. The results revealed that these simulations can help students to interpret most difficult materials they encounter in the social statistics course. To the teachers, the simulation probability tools like TinkerPlots could help prospective teachers in the prediction and related inference skills, which would influenced the students in a positive way (Koparan & Yilmaz, 2015).

Schenker (2007) examined the effectiveness of using various forms of technology in statistics instruction in higher education. He concluded from 46 studies that technology was modestly effective in enhancing students' statistics achievement, among which simulation was the most significantly effective technology type. Similarly, Hsu (2003) integrated the results from 25 empirical studies to examine the effectiveness of computer-assisted instruction in statistics. She found that expert systems and drill-and-practice programmes were the most effective technology types. Sosa, Berger, Saw, and Mary (2011) who analysed 45 experimental studies revealed that instructional time, students' level of education, and embedded assessment can serve as meaningful moderators in determining students' statistics achievement.

2.2. Augmented reality and its potential in education

AR, which brings learners a richer interactive experience than ordinary virtual learning environments, could help high school students understand the concepts and build it step by step. Some studies investigated the value of AR classrooms and showed that educational AR applications were largely positive. It could be a valuable teaching and learning tool at elementary education (Billinghurst & Duenser, 2012).

In recent years, AR applications in education have gained attention by many educational organisations and researchers. The mostly cited advantages of AR included encouraging motivation, comprehension and involvement of students (Coimbra, Cardoso, & Mateus, 2015), and reducing students' cognitive load (Wu, Hwang, Yang, & Chen, 2018). Further, AR could enhance the understanding and internalisation of learning contents. Many studies choose AR as technology to present virtual learning objects, but the teaching methods and knowledge gain were the challenges faced when applying AR in an educational setting (Coimbra et al., 2015).

Recent studies found that AR-based system could not only have positive results in learning attitudes but also achievements or performance, motivation and critical thinking abilities, and could encourage students to interact and practice more (Chang & Hwang, 2018).

However, for the topic of mathematics, Wetzel, Radtke, and Stern (1994) found that image-based teaching can aid students in focusing their attention, and research on the application of Augmented Reality in K-12 Mathematics Education has been performed. Because of its three-dimension properties, Researchers chose solid geometry as the mathematical area that would be integrated with AR. The effectiveness of learning solid geometry with AR has been assessed and proved in a learning system designed for high school students (Lin, Chen, & Chang, 2015). This research also found that low-achieving students developed a more positive attitude using an AR learning tools.

In addition to geometry, AR also has applications in algebra. Barraza Castillo, Cruz Sánchez, and Vergara Villegas (2015) posed a case study of mobile-based AR application in quadratic equations. Students could see and learn about the image of quadratic equation on their own table. Results showed the enhancement in teaching and learning process was taken by AR.

Notably, research on the application of Augmented Reality in K-12 Mathematics Education about geometry and functions has been performed based on the free 3D virtual world created with AR. However, studies have also showed that AR can be used to demonstrate or explain abstract concepts which were not easy to understand. Dünser, Walker, Horner, and Bentall (2012) introduced an AR-based book to teach electromagnetism concepts and the effectiveness in helping students learn was evaluated. The descriptive results of the difference in average percentage of correct answers given by two groups of participants showed that AR books have some potential in aiding the learning of complex spatial concepts. The complex and difficult learning content could be simplified by the AR-based learning tools (Chen, Huang, & Chou, 2017).

AR's effect on concept understanding was studied using a psychological framework of AR learning which was developed by researchers. This framework used three dimensions to explain AR's effect in teaching and learning process: physical, cognitive and contextual. It is worth noting that in the cognitive dimension, AR could help students to understand figurative language. In this process, AR aids scaffolding (Bujak et al., 2013).

For instance, in a traditional course on probability, teachers tend to instruct students to toss coins to explore the relations and differences between empirical probability and theoretical probability. This can be time-consuming and boring.

With the development of technology, some teachers may use animated videos built in Adobe Flash to simulate the coins flipping. However, then students feel the task is inauthentic. If we apply AR to simulate tossing coins, students may be more engaged in this experiment, and the data collecting process can be finished in a short time. Therefore, we decided to develop a unit that would teach probability, sample space and other key statistical concepts.

This paper aimed to present a new way for high school students, which were touching probability for the first time, to sense the probability by using AR. Three mobile AR-based mobile applications were developed to teach a basic statistical concept of probability at high school level. Tests, questionnaires and interviews were taken in this research.

3. Method

3.1. Participants and groups

All of 68 seventh-grade students were from junior high schools in an urban-rural fringe area. The participants' ages ranged from 13 to 15 years. All students were from the same class at the school, and they had similar background knowledge of the concepts of probability. The students were divided into groups of four for collaborative learning. The groups were further distributed into an experimental group and a control group. Both groups had similar average scores on the pre-tests such. In summary, the experimental group consisted of 33 participants, and the control group consisted of 35 participants. All participants were informed that their participation was voluntary, and they were assured that all responses and answers in this research would be kept completely confidential and would not affect their mathematics scores in school.

3.2. Instructional design

Figure 1 shows the teaching and learning structure of the probability unit for both groups. This structure was based on a mathematical textbook that is used widely in China. In each of those lessons, a unique Augmented Reality application was designed for the students in the experimental group to explore the probability concepts on their own or with their classmates, whereas the students in the control group undertook traditional learning activities. All three lessons lasted for six weeks.

The structure of each lesson is shown in Figure 2. The first 5 min of the lesson was a Do Now activity, a short activity on the board for students to work on as soon as they enter the lesson; group discussion was allowed throughout the process. In the next 15 min, students in both groups were introduced to the specific topic of that day. The format of this section of the lesson was a mixture of lecture and group discussion. For the next 15 min, the students explored the concepts of probability by using Augmented Reality applications on the tablet computers. The students were asked to record experimental data, and they could communicate and seek help from the instructor or the assistant as shown in Figure 3. Following the experiment was a 5-minute discussion in which students could exchange ideas and discoveries with other students. The last 5-minute

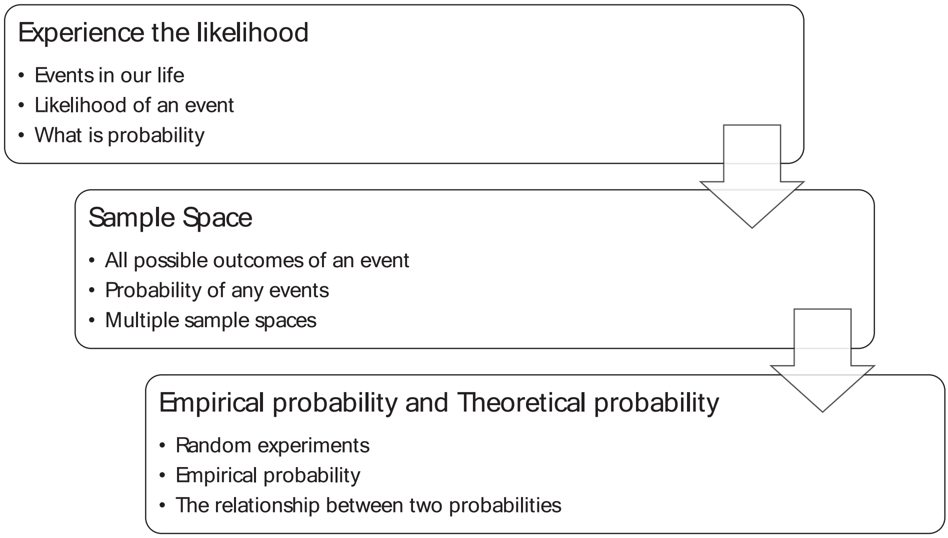

Figure 1. Structure of probability unit.

instruction was the summary guided by the instructor. This structure of lesson would be repeated once in each three different topics.

3.3. AR-based applications design

There were three AR applications designed for the students in their lessons. The AR-based learning application system's structure is shown in Figure 4. The physical device layer calls the camera of the Android device to capture a pixel image. The development layer provides the software development kit to help the application layer call the interface methods. The function module layer provides a graphical interface for users to use different AR functions.

The first application was designed for the lesson "Experience the likelihood" in Figure 1. It was a simple game for two students. One student started the application by placing the designed AR card in the view of camera. Once the camera captured the car, a dice appeared on the screen and was rolled. The result was recorded and shown on the screen. Then, the second student repeated the process, and the game went into the second round. In this round, students could choose to roll the die for a second time or stop the game if they believed that the numbers they had rolled were sufficient to beat their opponent. The game would continue to a next round until both students chose to stop it. Finally, the student who had seven points or was closest to seven points won the game. However, if a student had more than seven points, he or she would automatically lose. This application helped students sense the probability in the real world. As they played the

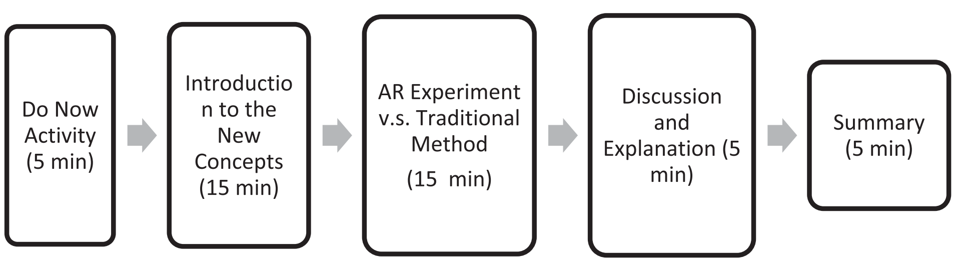

Figure 2. Structure of each lesson.

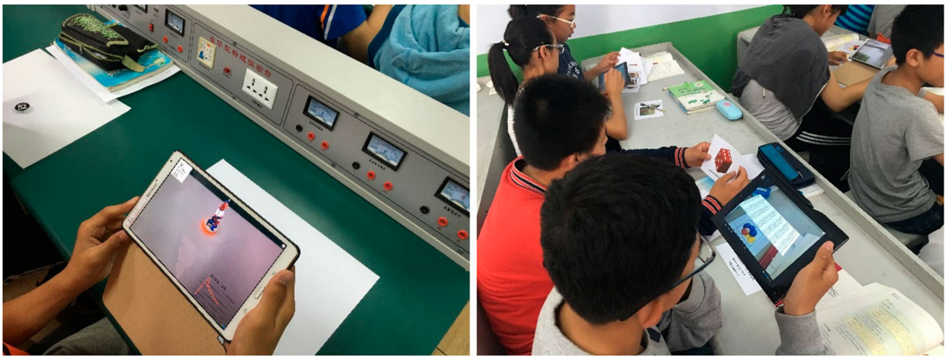

Figure 3. AR experiment in the lesson.

application, they should have gained an increasing understanding about when to continue and when to quit.

And the second application was designed for the lesson "Sample Space" in Figure 1. This application trains students to be familiar with sample spaces of equally likely probability events. In this application, students were divided into groups of two, and each group used one tablet computer. There were four types of paper designed for the application, termed coins, dice, cards and balls. Each of the papers represented one equally likely probability event. When students placed a

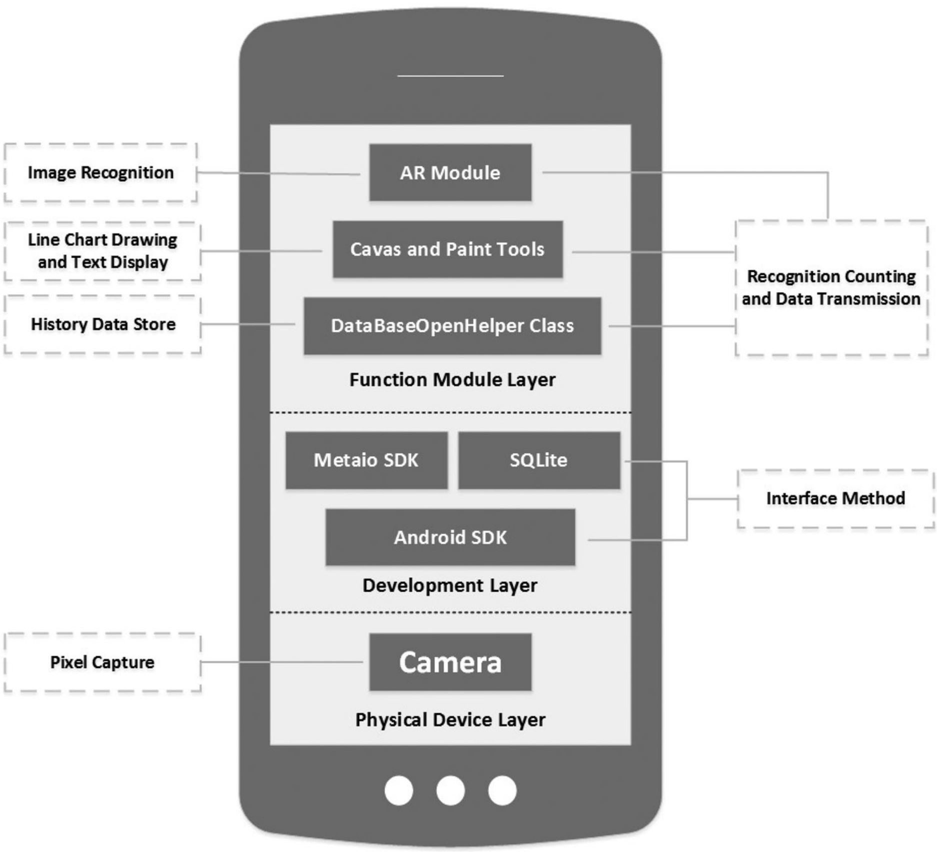

Figure 4. System structure of the application.

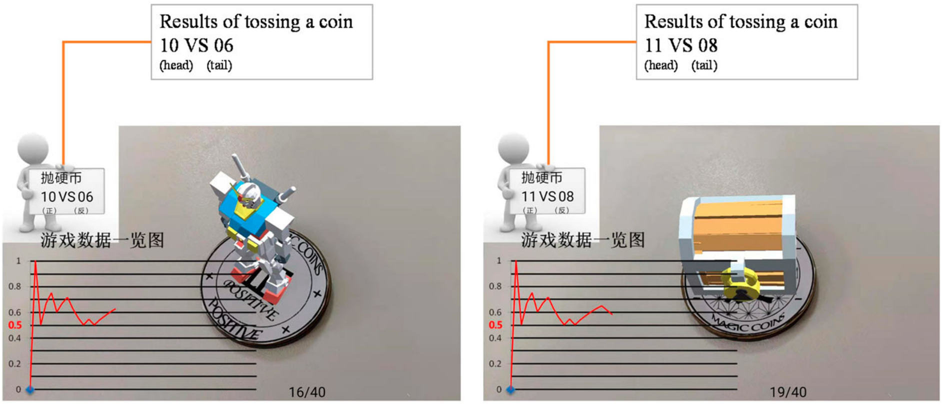

Figure 5. Recognition in AR applications.

paper in front of the camera, the entire sample space would appear on the screen. The students were asked to discuss the sample space of the probability event given on the paper with their teammates before checking the results on the tablet computers. In addition, students could simultaneously place two papers in front of the camera, so the resulting sample space would be a multiple sample space of two probability events.

And the third AR application was designed for the lesson "Empirical probability and Theoretical probability" (see Figure 1). When using this application, students should toss a real coin. Then the camera captures the head or tail side of the coin, and the corresponding 3D model is shown over the captured real scene. Meanwhile, the application tracks how many times the coin comes up heads or tails and displays the results in real time on the upper left corner of the screen. The curve graph of the head side probability is shown in real time on the lower left corner of the screen, as shown in Figure 5.

The third application could help students to save their time in the classroom environment for there is no need to collect the data and draw the curve graph by their own. In many using cases of AR it could bring more need of class time (Akçayır & Akçayır, 2017), this application goes on different ways of using AR in classroom: not for presenting something but for helping students' exploring studies.

When users exit the application, the historical data are recorded in the local database for users to access, and students can upload the data to the server, so the teachers can see all students' data

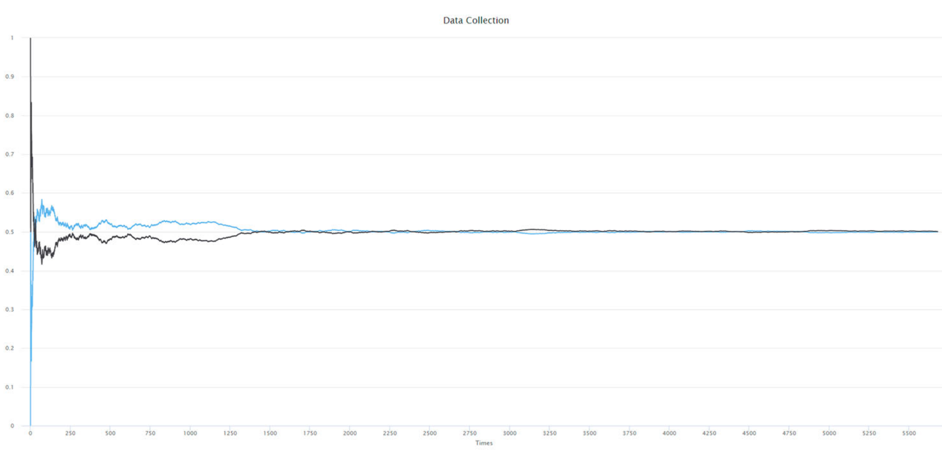

Figure 6. Data collection of tossing results in webpage.

through a website, as shown in Figure 6. This feature was based on the teachers' advice because it helped teachers explain the topic in this lesson, which could help students to understand the relationship between two types of probability, through their own produced data in classroom. In our study, a group could toss a physical coin about 200 times in a 15-minute learning activity. The teacher could get the results of about 3000 times of tossing a coin. These results could help students to see the process of empirical probability getting closer to the theoretical probability (0.5) as Figure 6 shows.

This data collection feature of the application could complete many more random experiments than the traditional experiment way (having students toss a real coin and record the data on their own).

3.4. Research design and tools

The experimental research was taken in this study. We chose tests, questionnaire and open-ended questions and interviews to explore the impacts of AR on students' learning gains and attitudes. The research hypothesis is as follow:

1. Students using AR in their learning could get more learning gains than those in control group.
2. Students' attitudes towards the AR application was positive and they are willing to use it in learning.

3.4.1. Tests: pre-test and post-test

The test we used to analyse the students' grasp of probability knowledge was the standard test distributed by the local Education Bureau to all local schools. We chose this standard test instead of creating a new test because this research would be more practical and meaningful if students could achieve better grades on the local tests.

However, in the pre-test, some professional terms were modified to easy-to- understand expressions for the benefit of students who may not have learned them.

Both tests consisted of 16 probability problems. The maximum score of each test was 100.

3.4.2. AR-based applications questionnaire

The questionnaire was derived from the technology acceptance questionnaire developed by Chu, Hwang, Tsai, and Tseng (2010). Cai, Wang, and Chiang (2014) and Cai, Chiang, Sun, Lin, and Lee (2017) modified the questionnaire to test the technology acceptance of AR tools. In this study, the questionnaire consists of three constructs: Satisfaction, Cognitive Validity and Cognitive Accessibility. This questionnaire was used to learn about the students' attitude, satisfaction and acceptance towards the mobile AR-based learning applications. This questionnaire was only administered with the post-test. Only students in the experimental group completed the questionnaire.

The Cronbach's Alpha coefficient for the questionnaire was 0.97, and the Cronbach's Alpha values of the three constructs are 0.96, 0.92 and 0.86, respectively. The results suggested that the questionnaire is reliable.

3.4.3. Open-ended questions and interviews

Besides the quantitative research tools, some of the students answered a few open-ended questions which were attached on the post-test. Researchers also interviewed the participating students face-to-face after the lessons.

Five open-ended questions were given to students in the experimental group after their post-test.

1. Which components in the lesson are the most enjoyable one to you?
2. What are the advantages of these AR applications in this unit?
3. What are the disadvantages of these AR applications in this unit?

4. What are the advantages of the traditional teaching and learning process in mathematics class (without tablet and AR)?
5. What are the disadvantages of the traditional teaching and learning process in mathematics class (without tablet and AR)?

These questions supplemented the questionnaire investigating students' attitudes to teaching and learning with AR applications.

Moreover, face-to-face interviews were carried out with four students who were randomly selected from the class about their opinions on AR instructions after the class. The questions included but were not limited to the following:

1. How would you compare the augmented reality teaching method to the traditional teaching method?
2. What mathematical knowledge didyou gain from the first mobile AR-based probability application (Experience the Likelihood)?
3. What mathematical knowledge did you gain from the second mobile AR-based probability application (Sample Space)?
4. What mathematical knowledge did you gain from the third mobile AR-based probability application (Tossing the Coins)?

The first question was selected because the focus of this study was to compare traditional mathematics instruction to the instruction with learning activities with AR application. The second, third and fourth questions are critical because academic achievement is important in determining the effectiveness of the study.

4. Experimental results

4.1. Comparing the learning gains of the experimental and control groups

The experiment produced 33*2 test results for the experimental group (33 for the pre-test and 33 for the post-test) and 35*2 for the control group. The maximum test score was 100. Table 1 shows the basic group descriptive statistics of pre-test and post-test scores.

Paired *t*-tests was conducted for the pre-test and post-test score variables of the experimental and control groups. Table 2 shows the results.

As seen in Table 2, students in both groups scored higher on the post-test than they did on the pre-test (experimental group: $t = 11.94$, $p < 0.01$, control group: $t = 3.06$, $p < 0.01$).

To check the effect of AR, we conducted a one-way analysis of covariance (ANCOVA) for the post-test scores of both groups. The covariances are students' pre-test scores (the scores before the treatment of AR), and the dependent variables are students' post-test scores (the scores after the treatment of AR). The results are shown in Table 3.

ANCOVA results revealed that experiment group obtained significantly higher scores in the post-test ($F = 30.48$, $p < 0.01$, $\eta 2 = 0.32$).

In summary, the students' learning gains in probability improved after the courses, and it is worth noting that the use of AR learning tools made the improvement more significant. Further, after the

Table 1. Descriptive statistics of test scores.

Group		N	Mean
pre-test score	experimental group	33	80.64
	control group	35	80.54
post-test score	experimental group	33	88.30
	control group	35	83.06

Table 2. Paired *t*-test for pre-test and post-test score variables of two groups.

Group		Mean	S.D.	t
post-test score minus pre-test score	experimental group	7.67	3.69	11.94*
	control group	2.51	4.86	3.06*

Table 3. Descriptive statistics of students' pre-test and post-test scores and ANCOVA summary.

		Before treatment		After treatment		Univariate ANCOVA			
Group	N	Mean	SD	Mean	SD	Mean (adjusted)	Standard error	*F*	eta^2
EG	33	80.64	11.25	88.30	10.00	88.26	.67	30.48*	.32
CG	35	80.54	10.52	83.06	9.31	83.09	.65		

courses, the difference between the groups increases, and the experimental group's score is higher than the control group's score. These results meet the hypothesis 1.

4.2. Descriptive statistics of AR-based applications questionnaire

All 33 students in the experimental group completed the questionnaire, and Table 4 shows the descriptive statistics of "Satisfaction".

The item "I like learning using AR" has the highest score (Mean = 4.06), which suggests that students would enjoy AR learning methods in other courses outside of mathematics. The high mean score of "I hope that other subjects or lessons such as physics and chemistry will apply AR applications to learning as well" supports it. The item "These AR-based applications can help me discover new problems and questions" has a high mean score, which suggests that this AR-based application helps students in discovering new questions in learning mathematics. AR fits well with lessons where students are encouraged to find and solve problems, so it is effective in mathematics classes. The mean score for all statements is higher than 3.00, which indicates that the students' satisfaction for these AR applications is mostly positive.

The descriptive statistics for items that measured the "Cognitive Validity" are shown in Table 5.

The statement "Using AR helps me to master important knowledge points I did not understand in the past." has the highest score, 4.01, which suggests that these applications help students in learning and understanding the content. This result supports the cognitive effect of AR in learning (Bujak et al., 2013).

Finally, the descriptive statistics for items that measured to the "Cognitive Accessibility" are shown in Table 6.

Table 4. Descriptive statistics for the "satisfaction" construct.

Item	Mean	Std. Deviation
AR-based learning applications are more interesting than the learning methods we used in the classroom previously.	3.84	1.01
These AR-based applications can help me discover new problems and questions.	3.94	.96
Using AR-based applications allows me to see mathematical concepts such as probability.	3.95	.91
I like learning mathematics using AR.	3.98	.91
I like learning using AR.	4.06	.95
I hope that other courses such as physics and chemistry will also use AR applications to help students learn.	3.98	1.00
If possible, I hope to use AR applications to learn mathematics in the future.	4.03	.90
I would like to recommend learning with AR to my friends and classmates.	3.92	1.00
I'm interested in using AR applications to learn.	3.94	.92
The content of these applications is closely related to "probability", which is an interesting topic to me.	3.81	1.06
AR-based learning tool allows me to learn not only on my own but also with my friends and classmates.	3.97	.93
The design of these applications is pleasing and authentic.	3.98	1.03
The colours of applications are appropriate, as it is attractive and does not distract me.	3.86	1.03
I think learning probability and statistics using AR is necessary.	3.91	1.02

Table 5. Descriptive statistics for the "cognitive validity" construct.

Item	Mean	Std. Deviation
I believe that AR make learning materials more detailed and understandable.	3.97	.98
I think that using AR is very helpful for learning mathematics.	3.90	.96
Using AR helps me to master important knowledge points I did not understand in the past.	4.01	.96
AR provides abundant space for me to think and try, which helps me in solving problems.	3.86	1.06

Table 6. Descriptive statistics for the "cognitive accessibility" construct.

Item	Mean	Std. Deviation
Operating AR applications is not difficult.	4.14	.91
Learning to use AR does not cost me too much time and energy.	4.02	.99
The content of and procedures for learning activity are clear and understandable.	4.19	.79
I can grasp how to operate AR within a very short time.	4.20	.81

All four statements in this table have a mean score above 4.00, which suggests that the AR-based applications are easy for the junior-high school students to use.

The results from three constructs show the students' attitude and willing of using AR in classroom are positive, which meet the hypothesis 2.

4.3. Results of open-ended questions

Five open-ended questions were asked to students in experimental group only, and all of the 33 students responded.

The first open-ended question asked students to specify the most enjoyable component in the unit. The whole class performed Do Now activities and used three augmented reality applications. Except for one student who refused to provide answers, 100% of students selected Augmented reality applications as the most enjoyable part of the class. Among the three augmented reality applications, students voted the third application the most popular and interesting one.

The second and third open-ended questions asked students to write down the advantages and disadvantages of the augmented reality component of the lesson. For the advantages, 84% of the students wrote that AR made the class more interesting and fascinating, 68% wrote that AR led to more efficient teaching and learning, and 52% wrote that AR increased the motivation of study. However, 40% reported that some students were distracted by the tablet computers at certain times during the class.

The fourth and fifth open-ended questions asked students to list the advantages and disadvantages of traditional mathematical teaching. A total of 38% of the students reported that traditional teaching led to improved exam results. This may be because education in mainland China is largely examination-oriented so some students thought the traditional teaching and learning process could help more in their exam. But all students reported that traditional math classes were boring and inflexible.

4.4. Results of face-to-face interviews

Four students were randomly selected from the experimental group and were asked some questions about their opinions on AR instructions.

When one of the students was asked "how do you perceive the difference between the traditional math courses and the augmented reality technology in today's instruction?", he replied:

> Compared to traditional instruction, where we observe the process of flipping coins, as conducted by teachers, or spend more time on recording data and graphing the curve, AR makes us participate in the whole process personally and focus more on the abstract mathematical concepts such that the outcomes would be more memorable to us.

Another student answered:

> The empirical probability graph that is created automatically by the program made the research outcomes more approachable to us. Compared to the traditional methods, which require a long time for students to draw graphs, the AR application enabled students to flip a large amount of coins and obtain the desired graph at a much quicker pace.

Further, all students said that the application of sample spaces provided them with a new way to imagine and understand the abstract mathematical concept sample space.

Regarding the mathematical knowledge students gained from the mobile AR-based probability applications, one student answered that the first application (experience the likelihood) gave him a sense of the abstract concept of probability. The application of tossing the coins aided him in learning about theoretical probability and empirical probability and the relationship between them. The application of sample spaces provided him with vivid and novel demonstrations of the abstract concept of sample spaces.

Finally, the researchers have interviewed the class's original math instructor. She responded as follows:

> The students are more enthusiastic and curious about the materials in class using AR compared to their reactions in regular math class, especially when you instantly collect data from all students and graph the empirical probability on the instructional screen; the outcomes were more impressive and understandable to all students.

5. Discussion and conclusion

The analysis of pre-test and post-test of two groups revealed that two groups had no difference in scores before this unit of lessons. After these lessons, students using the AR application in probability learning received higher learning gains than students in control group, as the results of ANCOVA presented. That conclusion was as the same line of previous studies (Hsiao, Chen, & Huang, 2012; Lin et al., 2015).

The quantitative analysis of students' learning gains was statistically significant, and the qualitative analysis (included students' answers to the open-ended questions on the questionnaire) illustrated a strong improvement in the engagement of secondary students in the mathematical learning process with instruction featuring Augmented Reality technology. Most of the students gave positive feedback regarding the AR applications, especially the tossing coins application. For the advantages of AR, many of them mentioned interesting, fascinating, higher motivation and more efficient learning, which supported the positive results from questionnaire.

During the face-to-face interview, one student stated that AR helps him participate in the whole process of exploring relationships between mathematics concepts. That was an evidence of AR's enhancement of engagement in classroom. The help of understanding of abstract concepts such as sample space was mentioned in the interview by more than one student. From a constructivist perspective, AR helped students to construct knowledge of statistics (Bujak et al., 2013).

In the experienced math teacher's view, it is obvious that the augmented reality technology enhanced the students' mathematical learning experience. By using AR-based applications, students could visualise the real coins that they flipped on the screens and could see the three-dimensional cartoon figures with animations on1` top of the coins. This feature made the repetitious procedure of flipping coins more enjoyable for secondary students in the learning process.

To sum up, this study aims to investigate the effects that mobile AR-based learning applications on probability have on junior high school students' mathematics learning. Three AR applications were designed to support the exploring activities in classroom, to help students to understand and learn abstract concepts in probability and statistics. The main innovation of our use of AR is using it in the exploring process rather than simply using it to present some visual objects. From the experimental results, it was found that these AR-based learning applications can help students achieve higher learning gains when learning mathematics. Moreover, from the results of the attitude questionnaire and the open-ended questions, the students' attitude towards applications is positive. The researchers also consulted the junior high school math instructor.

Further studies may broaden the audience of this type of AR applications and explore the effect of AR on more personal characteristics of students such as learning anxiety, not only on the learning gains and attitudes. Comparison between different instructional strategies with AR applications could be considered.

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